Scientific Notation

Why your wrist (or keyboard) will thank you for not writing all those zeros

Note: There are lots of numbers on this page that may be easier to read if the document is printed out.

A [ScientificNotationWorksheet+Answers>scientific notation worksheet](#) accompanies this lesson. Be sure to check it out!

Why Use Scientific Notation?

Scientific Notation was developed in order to easily represent numbers that are either very large or very small. Here are two examples of large and small numbers. They are expressed in decimal form instead of scientific notation to help illustrate the problem:

The Andromeda Galaxy (the closest one to our Milky Way galaxy) contains at least 200,000,000,000 stars.
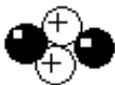
On the other hand, the weight of an alpha particle, which is emitted in the radioactive decay of Plutonium-239, is 0.000,000,000,000,000,000,000,000,006,645 kilograms.

As you can see, it could get tedious writing out those numbers repeatedly. So, a system was developed to help represent these numbers in a way that was easy to read and understand: Scientific Notation.

What is Scientific Notation?

Using one of the above examples, the number of stars in the Adromeda Galaxy can be written as:

2.0 x 100,000,000,000

It is that large number, 100,000,000,000 which causes the problem. But that is just a multiple of ten. In fact it is ten times itself eleven times:

10 x 10 x 10 x 10 x 10 x 10 x 10 x 10 x 10 x 10 x 10 = 100,000,000,000

A more convenient way of writing 100,000,000,000 is 10^{11}. The small number to the right of the ten is called the "exponent," or the "power of ten." It represents the number of zeros that follow the 1.

Though we think of zero as having no value, zeroes can make a number much bigger or smaller. Think about the difference between 10 dollars and 100 dollars. Any one who has balanced a checkbook knows that one zero can make a big difference in the value of the number. In the same way, 0.1 (one-tenth) of the US military budget is much more than 0.01 (one-hundredth) of the budget. (Though either one is probably more money than most of us will ever see in our checkbooks!)

So we would write 200,000,000,000 in scientific notation as:

2.0×10^{11}

This number is read as follows: "two point zero times ten to the eleventh."

How Does Scientific Notation Work?

As we said above, the exponent refers to the number of zeros that follow the 1. So:

$10^1 = 10$;
$10^2 = 100$;
$10^3 = 1,000$, and so on.

Similarly, $10^0 = 1$, since the zero exponent means that no zeros follow the 1.

Negative exponents indicate negative powers of 10, which are expressed as fractions with 1 in the numerator (on top) and the power of 10 in the denominator (on the bottom).

So:
$10^{-1} = 1/10$;
$10^{-2} = 1/100$;
$10^{-3} = 1/1,000$, and so on.

This allows us to express other small numbers this way. For example:

$2.5 \times 10^{-3} = 2.5 \times 1/1,000 = 0.0025$

Every number can be expressed in Scientific Notation. In our first example, 200,000,000,000 should be written as 2.0×10^{11}. In theory, it can be written as 20×10^{10}, but by convention the number is usually written as 2.0×10^{11} so that the lead number is less than 10, followed by as many decimal places as necessary.

It is easy to see that all the variations above are just different ways to represent the same number:

200,000,000,000 =
20×10^{10} ($20 \times 10,000,000,000$)
2.0×10^{11} ($2.0 \times 100,000,000,000$)
0.2×10^{12} ($.2 \times 1,000,000,000,000$)

This illustrates another way to think about Scientific Notation: the exponent will tell you how the decimal point moves; a positive exponent moves the decimal point to the right, and a negative one moves it to the left. So for example:

$4.0 \times 10^2 = 400$ (2 places to the right of 4);

while
$4.0 \times 10^{-2} = 0.04$ (2 places to the left of 4).

Note that Scientific Notation is also sometimes expressed as E (for exponent), as in 4 E 2 (meaning 4.0 x 10 raised to 2). Similarly 4 E -2 means 4 times 10 raised to -2, or $= 4 \times 10^{-2} = 0.04$. This method of expression makes it easier to type in scientific notation.

Sometimes the advantage of scientific notation is not immediately obvious. A number like 340 is much easier to read in decimal form than in scientific notation:

340 or 3.4×10^2?

What about the number 380,000? At the end of 1994 the Department of Energy's (DOE) inventory of high level radioactive waste was approximately 378,400 cubic meters.
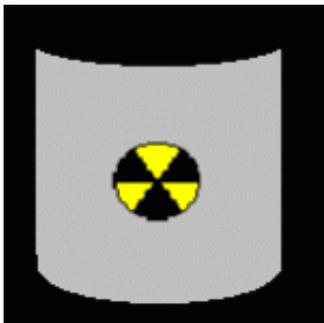
378,400 or 3.784×10^5?

The number 378,400 is also small enough to be readable. There are two reasons for expressing 378,400 in scientific notation rather than decimal form:

Computation: Scientific Notation makes adding, subtracting, multiplying and dividing numbers much simpler.
Creating and reading tables: Let's look at the table that this number came from:

Historical and projected cumulative volume of HLW stored in tanks, bins, and capsules, by site (Table 2.1 in the Department of Energy's Integrated Data Base Report-1994, DOE/RW-006, Rev. 11)End of Calendar Volum
Year

		Hanfo...
	1990	253.6
	1991	256.4
	1992	258.7
	1993	261.7
	1994	238.9

Look at Column 6. The label for that column has units of 10^3 m^3 (1,000 cubic meters). By making the units 10^3 m^3 rather than m^3 it is possible to use the number 378.4 instead of 378,400 in the table. This makes the graph much easier to read.

Addition and subtraction

The key to adding or subtracting numbers in Scientific Notation is to make sure the exponents are the same. For example,

$(2.0 \times 10^2) + (3.0 \times 10^3)$

can be rewritten as:
$(0.2 \times 10^3) + (3.0 \times 10^3)$

Now you just add $0.2 + 3$ and keep the 10^3 intact. Your answer is 3.2×10^3, or 3,200. We can check this by converting the numbers first to the more familiar form.

So:

$2 \times 10^2 + 3.0 \times 10^3 = 200 + 3,000 = 3,200 = 3.2 \times 10^3$

Let's try a subtraction example.
$(2.0 \times 10^7) - (6.3 \times 10^5)$

The problem needs to be rewritten so that the exponents are the same. So we can write
$(200 \times 10^5) - (6.3 \times 10^5) = 193.7 \times 10^5$,

which in Scientific Notation would be written 1.937×10^7.

Let's check by working it another way:
$2 \times 10^7 - 6.3 \times 10^5 = 20,000,000 - 630,000 = 19,370,000 = 1.937 \times 10^7$

Multiplication:

When multiplying numbers expressed in scientific notation, the exponents can simply be added together. This is because the exponent represents the number of zeros following the one. So:

10^1 x 10^2 = 10 x 100 = 1,000 = 10^3

Checking that we see: 10^1 x 10^2 = 10^{1+2} = 10^3

Similarly 10^1 x 10^{-3} = 10^{1-3} = 10^{-2} = 0.01

Again when we check we see that: 10 x 1/1000 = 1/100 = 0.01

Look at another example:
(4.0 x 10^5) x (3.0 x 10^{-1}).

The 4 and the 3 are multiplied, giving 12, but the exponents 5 and -1 are added, so the answer is:
12 x 10^4, or 1.2 x 10^5

Let's check:
(4 x 10^5) x (3 x 10^{-1}) = 400,00 x 0.3 = 123,000 = 1.2 x 10^5.

Interesting note: another way to see that 10^0 = 1 is as follows:
10^1 x 10^{-1} = 10^{1-1} = 10^0

It is also: 10 x 1/10 = 1

So 10^0 = 1

Division:

Let's look at a simple example:
(6.0 x 10^8) ÷ (3.0 x 10^5)

To solve this problem, first divide the 6 by the 3, to get 2. The exponent in the denominator is then moved to the numerator, reversing its sign. (Remember that little trick from your old math classes?) So we move the 10^5 to the numerator with a negative exponent, which then looks like this:
2 x 10^8 x 10^{-5}

All that's left now is to solve this as a multiplication problem, remembering that all you need to do for the "10^8 x 10^{-5}" part is to add the exponents. So the answer is:
2.0 x 10^3 or 2,000

Easy, huh? Well, even Dr. Egghead can't learn new concepts simply by reading them over. It takes a little practice. Lucky for you, we've put together a ScientificNotationWorksheet+Answers>worksheet on scientific notation! You'll be rattling off huge numbers like a pro in no time! Good luck!